Chapter 20.
Inference for General Causal Estimands

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Average effect may not be the most interesting value!

Example : Effect of labor market training program

- Policy makers interested in the effect on low-income people,
- or interested in effect on inequality in outcomes.

Causal estimands can be defined to be a general function:

$$\tau = \tau(\boldsymbol{Y}(0), \boldsymbol{Y}(1), \boldsymbol{X}, \boldsymbol{W})$$

(Example)

Average treatment effect (ATE) :

$$\tau_{\text{fs}} = \frac{1}{N} \sum_{i=1}^{N} (Y_i(1) - Y_i(0))$$

s^{th} quantile treatment effect :

$$\tau_{\text{quant}}^{s} = q_{Y(1)}^{s} - q_{Y(0)}^{s}, \quad q_{Y}^{s} = \inf_{q} \left\{ \frac{1}{N} \sum_{i=1}^{N} 1(Y_i \leq q) \geq s \right\}$$

Difference in standard deviations :

$$\tau_{\text{sd}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(Y_i(1) - \bar{Y}(1)\right)^2} - \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(Y_i(0) - \bar{Y}(0)\right)^2}$$

How to inference

In this chapter, we use model-based imputation :

1. Choose which model to use.
2. Sample model's parameters using MCMC.
3. Directly calculate causal estimand we consider.

(1) Choose which model to use

The Lalonde NSW Observational Job-Training Data :

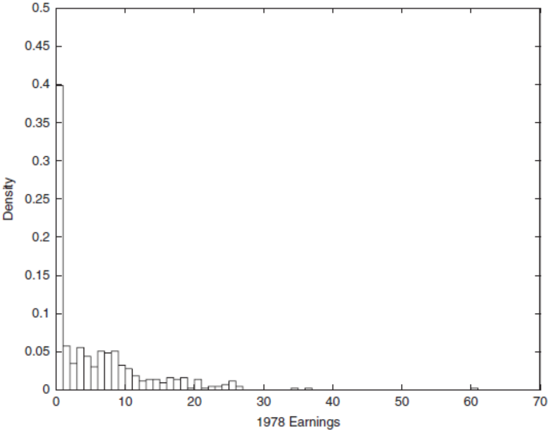

Figure 20.1. Histogram of 1978 earnings, trimmed Lalonde non-experimental data

(1) Choose which model to use

Model 1 : Single block with covariates

- $\Pr\left(Y_i(0) > 0 \mid X_i, \theta\right) = \frac{\exp(\gamma_{c,0}+X_i\gamma_{c,1})}{1+\exp(\gamma_{c,0}+X_i\gamma_{c,1})}$
- $\left(\ln Y_i(0) \middle| Y_i(0) > 0, X_i, \theta\right) \sim \mathcal{N}\left(\beta_{c,0} + X_i\beta_{c,1}, \sigma_c^2\right)$

- $\Pr\left(Y_i(1) > 0 \mid X_i, \theta\right) = \frac{\exp(\gamma_{t,0}+X_i\gamma_{t,1})}{1+\exp(\gamma_{t,0}+X_i\gamma_{t,1})}$
- $\left(\ln Y_i(1) \middle| Y_i(1) > 0, X_i, \theta\right) \sim \mathcal{N}\left(\beta_{t,0} + X_i\beta_{t,1}, \sigma_t^2\right)$

where $\theta = \left(\gamma_c, \gamma_t, \beta_c, \beta_t, \sigma_c^2, \sigma_t^2\right)$

Model 2 : Multiple blocks without covariates

- $B_i(j) = 1\left(b_{j-1} \leq \hat{e}\left(X_i\right) < b_j\right)$
- $\Pr\left(Y_i(0) > 0 \mid B_i(j) = 1, \theta\right) = \frac{\exp \gamma_c(j)}{1+\exp \gamma_c(j)}$
- $\left(\ln Y_i(0) \middle| Y_i(0) > 0, B_i(j) = 1, \theta\right) \sim \mathcal{N}\left(\beta_c(j), \sigma_c^2\right)$
- $\Pr\left(Y_i(1) > 0 \mid B_i(j) = 1, \theta\right) = \frac{\exp \gamma_t(j)}{1+\exp \gamma_t(j)}$
- $\left(\ln Y_i(1) \middle| Y_i(1) > 0, B_i(j) = 1, \theta\right) \sim \mathcal{N}\left(\beta_t(j), \sigma_t^2\right)$.

where $\theta = \left(\gamma_c(j), \gamma_t(j), \beta_c(j), \beta_t(j), j = 1, \ldots, J, \sigma_c^2, \sigma_t^2\right)$

(2) Sample model's parameters

- Draw values of the parameters from the posterior distribution given the observed data. (using MCMC)

(3) Calculate causal estimand

After Sampling model's parameters,

- Method 1 :
  1. Draw values for missing potential outcomes.
  2. Calculate the estimand as a function of observed and imputed potential outcomes.
- Method 2 :
  1. Draw values for both potential outcomes.
  2. Calculate the estimand as a function of imputed potential outcomes.
- Doing so (method 1 & 2) repeatedly gives us the draws from the posterior distribution of the causal estimand.